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Twisting warped supergravity

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Abstract: We study gauged five-dimensional supergravity on the interval $[0, \pi R]$. We find a set of boundary conditions with respect to which the theory is locally supersymmetric. For theories with detuned brane tensions ($\Lambda_4 < 0$), we show that these boundary conditions can be used to spontaneously break global supersymmetry. For the original, tuned Randall-Sundrum scenario ($\Lambda_4 = 0$), we prove that the locally supersymmetric boundary conditions are also globally supersymmetric. We lift the theory from $[0, \pi R]$ to S^1 and $\mathbb{R}$, with arbitrary twists for the fermions, and cast these results in the language of the Scherk-Schwarz mechanism.

Keywords: Field Theories in Higher Dimensions, Supersymmetry Breaking, Supergravity Models.

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1. Introduction

The supersymmetric Randall-Sundrum scenario [1]–[3] is based on five-dimensional supergravity on a manifold with boundary. The fifth dimension is the interval $[0, \pi R]$, usually realized as the orbifold $S^1/\mathbb{Z}_2$. The background is warped, with a nontrivial profile in the fifth dimension. The bulk contains pure supergravity with a cosmological constant $\Lambda_5 = -6\lambda^2$; the boundaries correspond to three-branes with tensions $T_0 = 6\lambda_0$ and $T_\pi = -6\lambda_\pi$.

In a recent paper [3], we constructed the locally supersymmetric action for the "detuned" case, in which T_0, T_π and Λ_5 are not related. We found that supersymmetry imposes boundary conditions on the bulk fields and requires $|\lambda_{0,\pi}| \leq \lambda$. The background was warped, with either a flat ($Mink_4$) or anti-de Sitter (AdS_4) metric on the four-dimensional slices. The theory was locally supersymmetric whenever the boundary conditions were satisfied. In this paper, we identify a subset of the boundary conditions for which supersymmetry is spontaneously broken.

We begin by considering warped supergravity in the "downstairs" picture, in which the fifth dimension is the interval $[0, \pi R]$. This approach is self-consistent and has the advantage that it avoids certain complications (discontinuous fields, delta-function singularities, etc.) that arise in the "upstairs" picture, on S^1 or $\mathbb{R}$. In the downstairs picture, the Scherk-Schwarz mechanism [4] is nothing but spontaneous supersymmetry breaking by boundary conditions. We will show that this breaking can be accomplished in every AdS_4 background, but never in the case of $Mink_4$.

We then lift $[0, \pi R]$ to the "upstairs" picture, in which the fifth dimension is S^1 or $\mathbb{R}$. We use a *broken* symmetry to construct a "twisted" lifting, in which the fermions rotate along the extra dimension, while the bosons remain periodic. The method is very general and can be used in other applications.

In the "upstairs" picture, the fields can twist and jump. There is freedom to interchange the twists and the jumps, maintaining the same boundary conditions on the fundamental domain. We emphasize that all liftings describe the same physics.

2. Working "downstairs"

2.1 Introduction

In this section we summarize the results from ref. [3], converting them to the "downstairs" picture, in which the fifth dimension is the interval $z \in [0, \pi R]$, rather than the circle S^1. We then derive a new result: in the AdS_4 warped background, supersymmetry can be spontaneously broken by a *continuous* family of boundary conditions. In the next section, we will see how the supersymmetry breaking can also be described by a Scherk-Schwarz twist.

2.2 Bulk action

The bulk action of gauged five-dimensional supergravity is

$$S_{\rm bulk} = \int d^5x e_5 \Big\{ -\frac{1}{2}R + \frac{i}{2}\widetilde{\Psi}^i_M \Gamma^{MNK} D_N \Psi_{Ki} - \frac{1}{4}F_{MN}F^{MN} + \\ + 6\lambda^2 - \frac{3}{2}\lambda\, \vec{q}\cdot\vec{\sigma}_i{}^j \widetilde{\Psi}^i_M \Sigma^{MN}\Psi_{Nj} + \cdots \Big\}. \tag{2.1}$$

The action depends on a unit vector $\vec{q} = (q_1, q_2, q_3)$ that defines a gauged $U_{\vec{q}}(1)$ subgroup of the $\mathrm{SU}(2)_R$ automorphism group. (The complete action, its supersymmetry transformations, and our conventions are collected in ref. [3].)

The action of an element $U \in \mathrm{SU}(2)_R$ on a symplectic Majorana spinor Ψ_i (and on its two-component constituents ψ_i) is given by

$$\Psi'_i = \widetilde{U}_i{}^j \Psi_j\,, \qquad \psi'_i = U_i{}^j \psi_j\,, \tag{2.2}$$

where $\widetilde{U} = \sigma_3 U \sigma_3$. The $\mathrm{SU}(2)_R$ symmetry is broken by the gauging. The symmetry is formally restored if we rotate the parameters as follows,

$$(\vec{q}\,'\cdot\vec{\sigma}) = \widetilde{U}(\vec{q}\cdot\vec{\sigma})\widetilde{U}^\dagger\,, \qquad (\vec{p}\,'\cdot\vec{\sigma}) = U(\vec{p}\cdot\vec{\sigma})U^\dagger\,. \tag{2.3}$$

Here $\vec{p}=(-q_1,-q_2,q_3)$ is defined by $(\vec{p}\cdot\vec{\sigma})=\sigma_3(\vec{q}\cdot\vec{\sigma})\sigma_3$. The unbroken $U_{\vec{q}}(1)$ subgroup is given by

$$\widetilde{U}=\exp(i\omega\vec{q}\cdot\vec{\sigma})\,,\qquad U=\exp(i\omega\vec{p}\cdot\vec{\sigma})\,. \tag{2.4}$$

2.3 Boundary conditions

Under (local) supersymmetry, variation of the action (2.1) gives rise a boundary term:

$$\delta S_{\rm bulk}=\int_{\mathcal{M}}d^5x(\partial_M K^M)=\int_{\partial\mathcal{M}}d^4x(n_M K^M)=\int_{z=\pi R}d^4xK^5-\int_{z=0}d^4xK^5\,. \tag{2.5}$$

In this section we present boundary conditions that make the boundary term vanish.

We start by imposing the simplest bosonic boundary conditions that support warped backgrounds. On the boundary at $z=0$, we take

$$e_m^{\hat{5}}=e_5^a=B_m=0\,,\qquad \omega_{ma\hat{5}}=\lambda_0 e_{ma}\,. \tag{2.6}$$

Demanding that these conditions be preserved under supersymmetry gives rise to the following fermionic boundary conditions:

$$\eta_2=\alpha_0\eta_1\,,\qquad \psi_{m2}=\alpha_0\psi_{m1}\,,\qquad \psi_{51}=-\alpha_0^*\psi_{52}\,. \tag{2.7}$$

Supersymmetry also requires that the parameters λ_0 and α_0 be related as follows,

$$\lambda_0=f(\alpha_0,\vec{q}\,)\lambda\,,\qquad f(\alpha,\vec{q}\,)=-\frac{(\alpha+\alpha^*)q_1+i(\alpha^*-\alpha)q_2+(\alpha\alpha^*-1)q_3}{1+\alpha\alpha^*}\,. \tag{2.8}$$

This relation couples the fermionic and bosonic boundary conditions. The boundary conditions decouple in the flat case, when $\lambda=0$.

For the boundary at $z=\pi R$, identical reasoning gives an analogous set of boundary conditions:

$$e_m^{\hat{5}}=e_5^a=B_m=0\,,\qquad \omega_{ma\hat{5}}=\lambda_\pi e_{ma}\,, \tag{2.9}$$

$$\eta_2=\alpha_\pi\eta_1\,,\qquad \psi_{m2}=\alpha_\pi\psi_{m1}\,,\qquad \psi_{51}=-\alpha_\pi^*\psi_{52}\,, \tag{2.10}$$

where

$$\lambda_\pi=f(\alpha_\pi,\vec{q}\,)\lambda\,. \tag{2.11}$$

The parameters α_0 and α_π are not related because the supersymmetry is local. The full set of boundary conditions forms a two-parameter family.

With the above boundary conditions, the action is invariant under $N=2$ supersymmetry, with arbitrary supersymmetry parameters $\eta_1(x,z)$ and $\eta_2(x,z)$ in the bulk, restricted only on the boundary:

$$\eta_2(x,0)=\alpha_0\eta_1(x,0)\,,\qquad \eta_2(x,\pi R)=\alpha_\pi\eta_1(x,\pi R)\,. \tag{2.12}$$

Note that in the "downstairs" picture, there is no boundary action. Brane actions appear only on the covering space, where the parameters $T_0=6\lambda_0$ and $T_\pi=-6\lambda_\pi$ play the role of brane tensions.

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2.4 Warped backgrounds

A bosonic background, consistent with the equations of motion and the boundary conditions, is given by

$$e_m^a = a(z)\hat{e}_m^a(x)\,, \qquad e_5^{\hat{5}} = 1\,, \qquad e_m^{\hat{5}} = e_5^a = B_m = B_5 = 0\,. \tag{2.13}$$

The warp factor $a(z)$ satisfies the equation

$$a'(z)^2 = \lambda^2(a(z)^2 - K^2)\,. \tag{2.14}$$

The boundary conditions follow from $\omega_{ma\hat{5}} = -(a'/a)\, e_{ma}$,

$$a'(0) = -\lambda_0\, a(0)\,, \qquad a'(\pi R) = -\lambda_\pi\, a(\pi R)\,. \tag{2.15}$$

With this ansatz, the four-dimensional veirbein $\hat{e}_m^a(x)$ solves Einstein's equations with cosmological constant $\Lambda_4 = -3\lambda^2 K^2$. The normalization condition $a(0) = 1$ fixes K and $a(z)$ uniquely.

Local supersymmetry requires $|\lambda_{0,\pi}| \leq \lambda$ (as follows from $|f(\alpha, \vec{q}\,)| \leq 1$), so there are three distinct cases:

1. $-\lambda < \lambda_\pi < \lambda_0 < \lambda$. This gives $a(z) = K\cosh(\lambda z - c_0)$, where

$$K = \sqrt{1 - \left(\frac{\lambda_0}{\lambda}\right)^2}\,, \qquad c_0 = \frac{1}{2}\log\left(\frac{\lambda + \lambda_0}{\lambda - \lambda_0}\right). \tag{2.16}$$

   The metric on the four-dimensional slices is AdS_4, with $\Lambda_4 < 0$; the distance R is fixed:

$$\lambda\pi R = \frac{1}{2}\log\left(\frac{\lambda + \lambda_0}{\lambda - \lambda_0}\cdot\frac{\lambda - \lambda_\pi}{\lambda + \lambda_\pi}\right). \tag{2.17}$$

2. $\lambda_\pi = \lambda_0 = \pm\lambda$. This is the Randall-Sundrum case, with $a(z) = \exp(\mp\lambda z)$, and flat four-dimensional slices, $Mink_4$, with $\Lambda_4 = 0$. The distance R is arbitrary.

3. $\lambda_\pi \neq \lambda_0 = \pm\lambda$. This is the case where one brane tension is tuned to the bulk cosmological constant, but the other is not. In this case there is no static background.

2.5 Supersymmetry breaking

A bosonic background is (globally) supersymmetric if there is a solution to the Killing spinor equations:

$$\delta\psi_{m1,2} = 0\,, \qquad \delta\psi_{51,2} = 0\,. \tag{2.18}$$

For the case at hand, they are

$$\begin{aligned} 2\hat{D}_m\eta_1 + i\omega_{ma\hat{5}}\sigma^a\overline{\eta}_2 + i\lambda\sigma_m(q_3\overline{\eta}_2 + q_{12}^*\overline{\eta}_1) &= 0\,, \qquad & 2\partial_5\eta_1 + \lambda(q_3\eta_1 - q_{12}^*\eta_2) &= 0\,,\\ 2\hat{D}_m\eta_2 - i\omega_{ma\hat{5}}\sigma^a\overline{\eta}_1 + i\lambda\sigma_m(q_3\overline{\eta}_1 - q_{12}\overline{\eta}_2) &= 0\,, \qquad & 2\partial_5\eta_2 - \lambda(q_3\eta_2 + q_{12}\eta_1) &= 0\,, \end{aligned} \tag{2.19}$$

where $q_{12} = q_1 + iq_2$. With the ansatz $\eta_{1,2} = \beta_{1,2}(z)\eta(x)$, the equations reduce to:

1. the four-dimensional Killing spinor equation for $\eta(x)$,

$$2\widehat{D}_m\eta + i\lambda g\widehat{e}_m{}^a\sigma_a\overline{\eta} = 0\,, \tag{2.20}$$

where $g \in \mathbb{C}$, $g^* = q_{12}\beta_1^2 - q_{12}^*\beta_2^2 + 2q_3\beta_1\beta_2$ and $gg^* = K^2$;

2. equations for the fermionic warp factors,

$$2\mathcal{B}' + \lambda(\vec{p}\cdot\vec{\sigma})\mathcal{B} = 0, \quad \mathcal{B} \equiv \begin{pmatrix}\beta_1\\ \beta_2\end{pmatrix} \quad \Leftrightarrow \quad \begin{cases} 2\beta_1' + \lambda(q_3\beta_1 - q_{12}^*\beta_2) = 0, \\ 2\beta_2' - \lambda(q_3\beta_2 + q_{12}\beta_1) = 0, \end{cases} \tag{2.21}$$

where $\vec{p} = (-q_1, -q_2, q_3)$; these equations ensure that g is a constant;

3. a relation between the bosonic and fermionic warp factors,

$$a(z) = \mathcal{B}^\dagger\mathcal{B} = \beta_1\beta_1^* + \beta_2\beta_2^*\,. \tag{2.22}$$

The Killing spinor equation (2.20) has solutions in the AdS_4 and $Mink_4$ backgrounds. Equations (2.21) for the fermionic warp factors are easy to solve,

$$\beta_i(z) = U(z)_i{}^j\beta_j(0)\,, \qquad U(z) = \exp\left(-\frac{\lambda}{2}(\vec{p}\cdot\vec{\sigma})z\right). \tag{2.23}$$

The solution shows that the Killing spinor is *twisted*. (Note that $U(z)$ commutes with the gauged $U_{\vec{q}}(1) \subset \mathrm{SU}(2)_R$, but $U(z) \notin \mathrm{SU}(2)_R$.)

Therefore we see that the Killing spinor equations can be solved in *all* the (static) warped backgrounds. However, a true Killing spinor must also satisfy the boundary conditions (2.12),

$$\beta_2(0) = \alpha_0\beta_1(0)\,, \qquad \beta_2(\pi R) = \alpha_\pi\beta_1(\pi R)\,. \tag{2.24}$$

Let us fix $q_3 = 1$ to simplify the discussion. (Other choices of $\vec{q}$ can be obtained by an $\mathrm{SU}(2)_R$ rotation.) Then

$$\beta_1(z) = \exp\left(-\frac{\lambda}{2}z\right)\beta_1(0)\,, \qquad \beta_2(z) = \exp\left(+\frac{\lambda}{2}z\right)\beta_2(0) \tag{2.25}$$

and the boundary conditions (2.24) require

$$\alpha_\pi = \alpha_0\exp(\lambda\pi R)\,. \tag{2.26}$$

When this condition is not satisfied, there is no Killing spinor and supersymmetry is spontaneously broken.

Consider now the set of boundary conditions consistent with local supersymmetry. For $q_3 = 1$, eqs. (2.8) and (2.11) imply

$$|\alpha_0|^2 = \frac{\lambda - \lambda_0}{\lambda + \lambda_0}\,, \qquad |\alpha_\pi|^2 = \frac{\lambda - \lambda_\pi}{\lambda + \lambda_\pi}\,. \tag{2.27}$$

(The same relations follow from eq. (2.22).) Equation (2.17) then implies

$$|\alpha_\pi| = |\alpha_0|\exp(\lambda\pi R)\,. \tag{2.28}$$

This relates the magnitudes $|\alpha_0|$ and $|\alpha_\pi|$, but does not determine the phases $\alpha_0 = |\alpha_0|e^{i\varphi_0}$ and $\alpha_\pi = |\alpha_\pi|e^{i\varphi_\pi}$. If the phases are the same, $\varphi_0 = \varphi_\pi$ $(+2\pi n,\ n \in \mathbb{Z})$, eq. (2.26) is satisfied and supersymmetry is not broken. Otherwise, eq. (2.26) cannot be satisfied; there is no Killing spinor and supersymmetry is spontaneously broken by the twisted boundary conditions.

2.6 Shift in the Kaluza-Klein spectrum

We conclude this section with a simple example which illustrates how the phases φ_0 and φ_π shift the masses for the fermionic Kaluza-Klein modes.

We set $q_3 = 1$, and take the ansatz $\psi_{n1,2} = b_{1,2}(z)\psi_n(x)$, where $\psi_n(x)$ satisfies the four-dimensional gravitino equation with mass parameter $m \in \mathbb{R}$,

$$\hat{\epsilon}^{mpnk}\hat{\sigma}_p\hat{D}_n\overline{\psi}_k + 2m\hat{\sigma}^{mn}\psi_n = 0\,. \tag{2.29}$$

The five-dimensional fermionic equations of motion give rise to the following equations for the warp factors $b_{1,2}(z) \in \mathbb{C}$:

$$b_1' + \left(\frac{3\lambda}{2} + \frac{a'}{a}\right) b_1 = \frac{m}{a} b_2^*\,, \qquad b_2' + \left(-\frac{3\lambda}{2} + \frac{a'}{a}\right) b_2 = -\frac{m}{a} b_1^*\,. \tag{2.30}$$

The mass quantization follows from the boundary conditions,

$$b_2(0) = \alpha_0 b_1(0)\,, \qquad b_2(\pi R) = \alpha_\pi b_1(\pi R)\,. \tag{2.31}$$

For the AdS_4 case, the bosonic warp factor is $a(z) = K\cosh(\lambda z - c_0)$. The brane tensions are given by

$$\lambda_0 = \lambda\tanh(c_0)\,, \qquad \lambda_\pi = -\lambda\tanh(\lambda\pi R - c_0)\,. \tag{2.32}$$

As discussed above, local supersymmetry fixes the absolute values of α_0 and α_π, but allows arbitrary complex phases φ_0 and φ_π:

$$\alpha_0 = e^{-c_0}e^{i\phi_0}\,, \qquad \alpha_\pi = e^{\lambda\pi R - c_0}e^{i\phi_\pi}\,. \tag{2.33}$$

For this example, we follow ref. [5] and take $\lambda_0 = -\lambda_\pi$ $(T_0 = T_\pi)$. In this case, $c_0 = \frac{1}{2}\lambda\pi R$.

We now assume that $\lambda\pi R \ll 1$. Equations (2.30) simplify as follows,

$$b_1'(y) + \frac{3}{2}b_1 = Mb_2^*\,, \qquad b_2'(y) - \frac{3}{2}b_2 = -Mb_1^*\,, \tag{2.34}$$

where $y = \lambda z - c_0$ and $M = m/(\lambda K)$. ($M = 1$ corresponds to the massless mode in the AdS_4 space with cosmological constant $\Lambda_4 = -3\lambda^2K^2$.) Equations (2.34) are easy to solve; the solutions fall into two classes, depending on whether $M < 1.5$ or $M > 1.5$. For $M \gg 1$, the solutions are

$$b_1(y) = A\cos(My) + B\sin(My)\,, \qquad b_2(y) = B^*\cos(My) - A^*\sin(My)\,. \tag{2.35}$$

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With our assumptions, we find $K = 1$, $\alpha_0 = e^{i\varphi_0}$ and $\alpha_\pi = e^{i\varphi_\pi}$. Setting $A = A_0 e^{i\vartheta_1}$ and $B = B_0 e^{i\vartheta_2}$, we cast the boundary conditions into the following form,

$$\begin{aligned}
&\sin\left(\frac{\varphi_0+\varphi_\pi}{2}+\vartheta_1+\vartheta_2\right)\cos\left(\frac{\varphi_0-\varphi_\pi}{2}\right)=0,\\
&\sin\left(\frac{\varphi_0+\varphi_\pi}{2}+\vartheta_1+\vartheta_2\right)\sin\left(\frac{\varphi_0-\varphi_\pi}{2}\right)=\sin\left(m\pi R\right)\cos\left(\vartheta_2-\vartheta_1\right),\\
&\cos\left(\frac{\varphi_0+\varphi_\pi}{2}+\vartheta_1+\vartheta_2\right)\sin\left(\frac{\varphi_0-\varphi_\pi}{2}\right)=\sin\left(m\pi R\right)\sin\left(\vartheta_2-\vartheta_1\right),\\
&\frac{A_0}{B_0}=\frac{\cos\left(\varphi_0+\vartheta_1+\vartheta_2\right)+\sin\left(m\pi R\right)\cos\left(\vartheta_2-\vartheta_1\right)}{\cos\left(m\pi R\right)}.
\end{aligned} \tag{2.36}$$

The solution, valid for any φ_0 and φ_π, is

$$\frac{A_0}{B_0}=1\,,\qquad \vartheta_2-\vartheta_1=\frac{\pi}{2}\,,\qquad \vartheta_1+\vartheta_2=-\frac{\varphi_0+\varphi_\pi}{2}+\pi j\,,\qquad m\pi R=\pi j+\frac{\varphi_0-\varphi_\pi}{2}\,. \tag{2.37}$$

The Kaluza-Klein mass for the j'th gravitino mode is given by

$$m_j=\frac{j}{R}+\frac{\varphi_0-\varphi_\pi}{2\pi R}\,,\qquad j\in\mathbb{Z}\,. \tag{2.38}$$

This formula is valid for $m_j \gg \lambda K \approx \lambda$. Because $\lambda R \ll 1$, the condition is satisfied for all modes, except, perhaps, the lightest one. Note that the mass shift depends only on the *phase difference*. This must always be true because one phase can be absorbed by a field redefinition.

3. Lifting "upstairs"

3.1 Introduction

In this section we show how to lift the previous construction to the "upstairs" picture, in which the fifth dimension is the circle S^1 or the line $\mathbb{R}$. Lifting to a covering space brings some technical and conceptual advantages. For example, a manifold without boundary allows one to neglect total derivatives, while a simply connected manifold allows one to avoid multi-valued fields.

3.2 General procedure

We start by describing our general lifting procedure, using a symmetry that is broken when it acts only on fields, but is intact when the parameters also rotate. (For the case at hand, we use the $SU(2)_R$ symmetry, broken by the gauged U(1) subgroup.) The basic idea is to combine a broken symmetry transformation with a group motion on the covering space, choosing appropriate parameters on the different domains.

To see how this works, let us consider an action for fields $\Phi(x)$ on a space $\mathcal{M}$ with a set of parameters Q:

$$S=\int_{\mathcal{M}} dx\mathcal{L}[\Phi(x),Q]\,. \tag{3.1}$$

We assume that S is invariant under $G = \{\widehat{g}_i\}$, a discrete group of transformations that acts on Φ, $\mathcal{M}$ and Q,

$$\widehat{g}_i S = \int_{\widehat{g}_i(\mathcal{M})} dx' \mathcal{L}[\Phi'(x'), \widehat{g}_i Q] = \int_{\mathcal{M}} dx \mathcal{L}[\Phi(x), Q] = S\,. \tag{3.2}$$

(Here $x' = \widehat{g}_i x$, $\Phi' = \widehat{g}_i \Phi$; $\widehat{g}_i$ is a conventional symmetry if $\widehat{g}_i Q = Q$.) We also assume that the group action splits $\mathcal{M}$ into a set of disjoint subspaces, such that

$$\mathcal{M} = \cup \mathcal{M}_i\,, \qquad \mathcal{M}_i = \widehat{g}_i \mathcal{M}_0\,, \tag{3.3}$$

where $\mathcal{M}_0 \simeq \mathcal{M}/G$ is the fundamental domain.

We now wish to construct an action that is invariant under G, with the group acting on Φ and $\mathcal{M}$, but *not* on the parameters Q. Since

$$\widehat{g}_i \int_{\mathcal{M}_0} dx \mathcal{L}[\Phi(x), Q] = \int_{\widehat{g}_i(\mathcal{M}_0)} dx' \mathcal{L}[\Phi'(x'), Q] = \int_{\mathcal{M}_i} dx \mathcal{L}[\Phi(x), \widehat{g}_i^{-1} Q]\,, \tag{3.4}$$

the action

$$\widetilde{S} = \sum_i \int_{\mathcal{M}_i} \mathcal{L}[\Phi(x), Q_i]\,, \tag{3.5}$$

with $Q_i = \widehat{g}_i^{-1} Q$, describes a G-invariant theory on the covering space $\mathcal{M}$.

Now suppose that we restrict the fields $\Phi(x)$ to G-invariant configurations

$$\widehat{g}_i \Phi(\widehat{g}_i x) = \Phi(x)\,. \tag{3.6}$$

In this case the theory on the covering space is equivalent to the theory on the fundamental domain, $\mathcal{M}_0$. Only the fundamental domain $\mathcal{M}_0$ is physical; all other domains are its "mirror images". The space $\mathcal{M}_0 \simeq \mathcal{M}/G$ is, in general, an orbifold.

3.2.1 Example: supergravity on $S^1/\mathbb{Z}_2$

To illustrate the lifting procedure, we lift supergravity from the fundamental domain $\mathcal{M}_0 = [0, \pi R]$ to its covering space $\mathcal{M} = S^1$. Even in this simplest case, the lifting relies on a broken symmetry. This is the origin of the so-called "odd bulk mass term" in the supersymmetric Randall-Sundrum scenario.

We use the discrete group $G = \mathbb{Z}_2$, generated by the parity transformation $\mathbf{P}$. The group acts on the fifth coordinate ($x^5 = z$) and on the fields and supersymmetry parameters according to $\mathbf{P}z = -z$ and $\mathbf{P}\Phi = P(\Phi)\Phi$, where

$$\begin{aligned} P(e_m^a, e_5^{\hat{5}}, B_5, \psi_{m1}, \psi_{52}, \eta_1) &= +1\,, \\ P(e_m^{\hat{5}}, e_5^a, B_m, \psi_{m2}, \psi_{51}, \eta_2) &= -1\,. \end{aligned} \tag{3.7}$$

When $q_3 \neq 0$ and $\mathcal{M} = S^1$ ($z \in [-\pi R, \pi R]$), the action (2.1) is not invariant under $\mathbf{P}$. However, the action *is* invariant if $\mathbf{P}$ acts on $\vec{q}$ as follows,

$$\mathbf{P}(q_1, q_2, q_3) = (q_1, q_2, -q_3)\,. \tag{3.8}$$

This allows us to construct an invariant action following the procedure described in the previous section. The lifted action is just the bulk action (2.1), with q_3 having a different sign on each side of the circle:

$$\widetilde{S}_{\text{bulk}} = \int_{-\pi R}^{0} dz \int d^4x \mathcal{L}_{\text{bulk}}[-q_3] + \int_{0}^{\pi R} dz \int d^4x \mathcal{L}_{\text{bulk}}[q_3] \,. \tag{3.9}$$

The parameter $\vec{q}$ multiplies a gravitino bilinear, so q_3 is responsible for the "odd bulk mass term."

In ref. [3] we found that warped backgrounds also require a brane action,

$$\begin{aligned} S_{\text{brane}} = & \int d^5x e_4(-6\lambda_0 - 2\alpha_0(\psi_{m1}\sigma^{mn}\psi_{n1} + \text{h.c.}))\ \delta(z) + \\ & + \int d^5x e_4(+6\lambda_\pi + 2\alpha_\pi(\psi_{m1}\sigma^{mn}\psi_{n1} + \text{h.c.}))\ \delta(z - \pi R)\,, \end{aligned} \tag{3.10}$$

for arbitrary $\vec{q}$ in the bulk. The brane action gives rise to jumps and cusps in the fields. The total bulk-plus-brane action, the sum of (3.9) and (3.10), is supersymmetric, provided $\lambda_{0,\pi} = f(\alpha_{0,\pi}, \vec{q})\lambda$, where $f(\alpha, \vec{q})$ is defined in eq. (2.8). In general, the supersymmetry transformation for ψ_{52} must also be modified,

$$\delta\psi_{52} = \delta\psi_{52}\Big|_{\text{old}} - 4(\alpha_0\delta(z) + \alpha_\pi\delta(z - \pi R))\eta_1\,, \tag{3.11}$$

in which case the supersymmetry algebra closes and $\delta\psi_{52}$ is finite on the branes (see ref. [3]).

The $S^1/\mathbb{Z}_2$ approach enjoys a few technical and conceptual advantages:

1. Total derivatives can be dropped when checking invariance of the action under supersymmetry.
2. The boundary conditions given in section 2.2 follow from the action principle. (The jumps are induced by the brane action; together with the parity assignments they imply the boundary conditions.)
3. The parameters $\lambda_{0,\pi}$ and $\alpha_{0,\pi}$ acquire clear physical interpretations as brane tensions and brane localized mass terms, respectively.

We emphasize, though, that all physical statements established on the fundamental domain remain unchanged on the covering space. The construction introduces mirror images of spacetime that do not change the boundary conditions.

3.3 Scherk-Schwarz twisting

In this section, we use a generalization of the previous construction to lift $\mathcal{M}_0 = [0, \pi R]$ to $\mathcal{M} = \mathbb{R}$, with periodic bosonic fields and twisted fermionic fields, along the lines of Scherk and Schwarz [4]. We assume that the bosonic background is warped. The warp factor $a(z)$ does not satisfy $a(0) = a(\pi R)$, so we cannot use a simple translation of the bosonic fields. Instead, we use a set of parity reflections to make $a(z)$ continuous along $\mathbb{R}$. We construct twisted embeddings by choosing different parity operators at the ends of the interval $[0, \pi R]$.

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3.3.1 Twisted parity

In section 3.2.1 we defined the standard parity transformation $\mathbf{P}$. We now consider a more general parity transformation $\mathbf{P}_{tw}$, one that includes an $\mathrm{SU}(2)_R$ rotation of fermions. Its action on the fifth coordinate, bosonic fields and fermionic fields is, respectively,

$$\mathbf{P}_{tw} z = -z\,, \qquad \mathbf{P}_{tw}\phi = P(\phi)\phi\,, \qquad \mathbf{P}_{tw}\Psi = UZ\Psi\,, \tag{3.12}$$

where $U \in \mathrm{SU}(2)_R$, $Z = \sigma_3 = \left(\begin{smallmatrix}1 & 0\\ 0 & -1\end{smallmatrix}\right)$ and $\Psi = \left(\begin{smallmatrix}\psi_{m1}\\ \psi_{m2}\end{smallmatrix}\right)$ (and likewise for ψ_5 and η). The requirement $\mathbf{P}_{tw}\mathbf{P}_{tw} = \mathbf{1}$ implies

$$(UZ)^2 = 1 \quad \Leftrightarrow \quad ZUZ = U^{-1}\,. \tag{3.13}$$

We denote by $\mathcal{S}$ the set of U satisfying this condition. Note that $\mathcal{S}$ is *not* a group.

A general element $U \in \mathrm{SU}(2)_R$ can be written as follows,

$$U = u_0 + i\vec{u}\cdot\vec{\sigma} = \begin{pmatrix} u_{03} & u_{21}\\ -u_{21}^* & u_{03}^*\end{pmatrix}, \qquad |u_{03}|^2 + |u_{21}|^2 = 1\,. \tag{3.14}$$

(We use notation $u_{ab} = u_a + iu_b$.) For the set $\mathcal{S}$, we find

$$U \in \mathcal{S} \quad \Leftrightarrow \quad \sigma_3 U \sigma_3 = U^{-1} = U^\dagger \quad \Leftrightarrow \quad u_3 = 0\,. \tag{3.15}$$

Taking $u_0 = \cos\theta$ and $u_{21} = e^{i\phi}\sin\theta$, we can parametrize $\mathcal{S}$ as follows,

$$U(\theta,\phi) = \begin{pmatrix} \cos\theta & e^{i\phi}\sin\theta\\ -e^{-i\phi}\sin\theta & \cos\theta\end{pmatrix}. \tag{3.16}$$

The $\mathrm{SU}(2)_R$ symmetry is broken by the vector $\vec{q}$. Only transformations in $U_{\vec{q}}(1)$,

$$U = \exp(i\omega\vec{p}\cdot\vec{\sigma}) = \cos\omega + i(\vec{p}\cdot\vec{\sigma})\sin\omega\,, \tag{3.17}$$

with $\vec{p} = (-q_1, -q_2, q_3)$, leave the action invariant.[1] Invariance under *any* $U \in \mathrm{SU}(2)_R$ is achieved if we rotate $\vec{q}$,

$$\mathbf{U}\vec{q} = \vec{q}\,'\,, \qquad (\vec{p}\,'\cdot\vec{\sigma}) = U(\vec{p}\cdot\vec{\sigma})U^\dagger\,. \tag{3.18}$$

Since $\mathbf{P}_{tw} = \mathbf{UP}$, this determines the action of $\mathbf{P}_{tw}$ on $\vec{q}$.

3.3.2 Bulk action

The bulk action depends on fields lifted from the fundamental domain $\mathcal{M}_0 = [0, \pi R]$ to the covering space $\mathcal{M} = \mathbb{R}$. We use *different* parity identifications across the branes at $z = 0$ and $z = \pi R$,

$$\begin{aligned}\Psi(-z) &= U_0 Z\Psi(z)\,,\\ \Psi(\pi R - z) &= U_1 Z\Psi(\pi R + z)\,,\end{aligned} \tag{3.19}$$

[1] When $q_3 \neq 0$, $U = \pm 1$ are the only *unbroken* symmetry transformations that satisfy eq. (3.13). These are the transformations that underlie the "flipped twisting" of refs. [5]–[7].

with $U_{0,1} \in \mathcal{S}$. Together, these imply

$$\Psi(z + 2\pi R) = U_\Delta \Psi(z)\,, \qquad U_\Delta = U_1 U_0^{-1}\,. \tag{3.20}$$

In this construction, the bosonic fields are periodic under shifts by $2\pi R$, but the fermionic fields twist by $U_\Delta \in \mathrm{SU}(2)_R$.

The twists U_0 and U_1 define a lifting of the bulk action (2.1) from $[0, \pi R]$ to $\mathbb{R}$ as in (3.5). The parameters $\vec{q}_{(n)} = (q_1, q_2, q_3)$ on the intervals $\mathcal{M}_n = \big(n\pi R, (n+1)\pi R\big)$ are given by

$$\left.\begin{pmatrix} -q_3 & q_{12}^* \\ q_{12} & q_3 \end{pmatrix}\right|_{\mathcal{M}_n} = \begin{cases} (U_\Delta)^{-k} \begin{pmatrix} -q_3 & q_{12}^* \\ q_{12} & q_3 \end{pmatrix} (U_\Delta)^k\,, & n = 2k\,, \\ (U_\Delta)^{-k} U_0^{-1} \begin{pmatrix} q_3 & q_{12}^* \\ q_{12} & -q_3 \end{pmatrix} U_0 (U_\Delta)^k\,, & n = 2k-1\,. \end{cases} \tag{3.21}$$

With this choice of parameters, the bulk action is invariant under translations by $2\pi R$, provided the fermions transform as

$$\Psi(z) \to \Psi'(z + 2\pi R) = U_\Delta \Psi(z)\,, \tag{3.22}$$

and the bosonic fields are trivially shifted, $\phi'(z + 2\pi R) = \phi(z)$. Note that the parameters $\vec{q}_{(n)}$ are *not* rotated.

3.3.3 Brane action

The parities for the mirror images of the two physical branes are given by[2]

$$\Psi(n\pi R - z) = U_n Z \Psi(n\pi R + z)\,, \qquad U_n = (U_\Delta)^n U_0\,. \tag{3.23}$$

Note that the two-component spinors ψ_{m1} and ψ_{m2} are *not* of definite parity. However, after the redefinitions

$$\widehat{\Psi}^{(n)}(z) = U_n^{-1/2} \Psi(z)\,, \tag{3.24}$$

the fields $\widehat{\psi}^{(n)}_{m1}$ and $\widehat{\psi}^{(n)}_{m2}$ are even and odd, respectively, across the n'th brane, because

$$\widehat{\Psi}^{(n)}(n\pi R - z) = Z \widehat{\Psi}^{(n)}(n\pi R + z)\,. \tag{3.25}$$

Using the parametrization $U_n = U(\theta_n, \phi_n)$, we find

$$\begin{aligned} \widehat{\psi}^{(n)}_{m1} &= \psi_{m1} \cos\left(\frac{1}{2}\theta_n\right) - \psi_{m2} e^{i\phi_n} \sin\left(\frac{1}{2}\theta_n\right)\,, \\ \widehat{\psi}^{(n)}_{m2} &= \psi_{m1} e^{-i\phi_n} \sin\left(\frac{1}{2}\theta_n\right) + \psi_{m2} \cos\left(\frac{1}{2}\theta_n\right)\,. \end{aligned} \tag{3.26}$$

As in ref. [3], the odd fermionic fields can be discontinuous across the branes. The jumps are determined by brane actions for the fermi fields. Using the fields of definite

[2]With $U_{0,1} \in \mathcal{S}$, one can show that $U_n \in \mathcal{S}$, even though $U_\Delta \notin \mathcal{S}$ in general. Any power of U_n is well-defined and belongs to $\mathcal{S}$ because $U(\theta, \phi)^w = U(w\theta, \phi)$.

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parity, we can simply copy the results of ref. [3] (see section 3.2.1). The complete brane action is just the sum of the individual brane actions,

$$S_{\rm brane} = \sum_{n\in\mathbb{Z}} \int d^5x e_4 \left(-6\lambda_n - 2\widehat{\alpha}_n \left(\widehat{\psi}^{(n)}_{m1}\sigma^{mn}\widehat{\psi}^{(n)}_{n1} + {\rm h.c.}\right)\right)\delta(z-n\pi R)\,. \tag{3.27}$$

It is easy to rewrite this action in terms of the original fields ψ_{m1} and ψ_{m2}, using eq. (3.26).

The brane action implies the boundary conditions

$$\widehat{\psi}^{(n)}_{m2} = \pm\widehat{\alpha}_n\widehat{\psi}^{(n)}_{m1}\,, \qquad \omega_{ma\hat{5}} = \pm\lambda_n e_{ma}\,, \tag{3.28}$$

with all fields evaluated at $z_n^\pm = n\pi R\pm 0$. In terms of the original fields ψ_{m1} and ψ_{m2}, the boundary conditions are

$$\psi_{m2}(z_n^\pm) = \alpha_n^\pm\psi_{m1}(z_n^\pm)\,, \qquad \alpha_n^\pm = \frac{\pm\widehat{\alpha}_n - e^{-i\phi_n}\tan(\frac{1}{2}\theta_n)}{1\pm\widehat{\alpha}_n e^{i\phi_n}\tan(\frac{1}{2}\theta_n)}\,. \tag{3.29}$$

Local supersymmetry requires $\lambda_n = f(\alpha_n^+, \vec{q}_{(n)})\lambda$, where $f(\alpha,\vec{q})$ is defined in eq. (2.8) and $\vec{q}_{(n)}$ is given in eq. (3.21).

3.3.4 Jumps, twists and boundary conditions

In terms of the spinors $\Psi = (\psi_{m1},\psi_{m2})^T$, the discontinuities across the branes can be parametrized as

$$\widehat{\Psi}^{(n)}(n\pi R-0) = \overline{U}_n\widehat{\Psi}^{(n)}(n\pi R+0)\,, \tag{3.30}$$

where $\overline{U}_n\in\mathcal{S}$. This gives rise to the following boundary conditions for the fermi fields,

$$\left(\overline{U}_n^{\pm1} - Z\right)\widehat{\Psi}^{(n)}(n\pi R\pm 0) = 0\,. \tag{3.31}$$

In general, the condition $(U-Z)\Psi = 0$ implies $\psi_{m2} = \mathcal{A}(U)\psi_{m1}$, where

$$\mathcal{A}(U) \equiv \frac{1-u_{03}}{u_{21}} = \frac{u_{21}^*}{1+u_{03}^*}\,. \tag{3.32}$$

(Consistency requires $u_3 = 0$, so $U\in\mathcal{S}$.) For the hatted fields, eq. (3.31) implies

$$\widehat{\alpha}_n = \mathcal{A}(\overline{U}_n)\,. \tag{3.33}$$

Analogously, in terms of the original fields, we find

$$\left(U_n^{-1/2}\overline{U}_n^{\pm1}U_n^{-1/2} - Z\right)\Psi(n\pi R\pm 0) = 0\,, \tag{3.34}$$

and therefore

$$\alpha_n^\pm = \mathcal{A}(U_n^{-1/2}\overline{U}_n^{\pm1}U_n^{-1/2})\,. \tag{3.35}$$

For $U = U(\theta,\phi)$, the parameters θ and ϕ can be read from $\mathcal{A}(U)$ as follows:

$$\mathcal{A}(U) = e^{-i\phi}\tan\left(\frac{1}{2}\theta\right). \tag{3.36}$$

Because there are just two physical branes, only quantities with $n = 0$ and 1 are independent. The jump matrices $\overline{U}_n$ obey the relations

$$U_n^{1/2}\overline{U}_n U_n^{-1/2} = \begin{cases} U_\Delta^k \overline{U}_0 U_\Delta^{-k}\,, & n = 2k\,, \\ U_\Delta^k \overline{U}_1 U_\Delta^{-k}\,, & n = 2k+1\,. \end{cases} \tag{3.37}$$

For a fixed set of twists (U_0, U_1), the matrices $\overline{U}_0$ and $\overline{U}_1$ are determined by the boundary conditions on the fundamental interval,

$$\psi_{m2}(+0) = \alpha_0 \psi_{m1}(+0)\,, \qquad \psi_{m2}(\pi R - 0) = \alpha_\pi \psi_{m1}(\pi R - 0)\,. \tag{3.38}$$

We find

$$\alpha_0 = \alpha_0^+ = \mathcal{A}(U_0^{-1/2}\overline{U}_0 U_0^{-1/2})\,, \qquad \alpha_\pi = \alpha_1^- = \mathcal{A}(U_1^{-1/2}\overline{U}_1^{-1} U_1^{-1/2})\,. \tag{3.39}$$

There is freedom to choose the twists and jumps, so long as the boundary conditions on the fundamental interval remain unchanged (see also refs. [8, 9]). In particular, we can trade twists for jumps, and vice versa:

1. Continuous twisted fields (no jumps, $\widehat{\alpha}_n = 0$): $\overline{U}_0 = 1$, $\overline{U}_1 = 1$

$$\alpha_0 = \mathcal{A}(U_0^{-1}) = -e^{-i\phi_0}\tan\left(\frac{1}{2}\theta_0\right), \qquad \alpha_\pi = \mathcal{A}(U_1^{-1}) = -e^{-i\phi_1}\tan\left(\frac{1}{2}\theta_1\right). \tag{3.40}$$

2. Jumping periodic fields (no twist, $U_\Delta = 1$): $U_0 = 1$, $U_1 = 1$

$$\alpha_0 = \mathcal{A}(\overline{U}_0) = e^{-i\overline{\phi}_0}\tan\left(\frac{1}{2}\overline{\theta}_0\right), \qquad \alpha_\pi = \mathcal{A}(\overline{U}_1^{-1}) = -e^{-i\overline{\phi}_1}\tan\left(\frac{1}{2}\overline{\theta}_1\right). \tag{3.41}$$

The physics is the same for each case.

3.3.5 Example

In section 2.6 we derived the Kaluza-Klein spectrum for the gravitini $\psi_{m1,2} = b_{1,2}^{(j)}(z)\psi_m^{(j)}(x)$, in the case where

$$q_3 = 1\,, \qquad \lambda_1 = -\lambda_2\,, \qquad \lambda\pi R \ll 1\,, \qquad m_j \gg \lambda\,. \tag{3.42}$$

We set $\alpha_0 = e^{i\varphi_0}$, $\alpha_\pi = e^{i\varphi_\pi}$, and found that the warp factors on the fundamental interval, $z \in [0, \pi R]$, are given by

$$\begin{aligned} b_1^{(j)}(z) &= A_0^{(j)} \exp\left\{+i\left(m_j z - \frac{\varphi_0}{2}\right)\right\}, \\ b_2^{(j)}(z) &= A_0^{(j)} \exp\left\{-i\left(m_j z - \frac{\varphi_0}{2}\right)\right\}, \end{aligned} \tag{3.43}$$

where

$$m_j = \frac{j}{R} + \frac{\varphi_0 - \varphi_\pi}{2\pi R} \tag{3.44}$$

is the Kaluza-Klein mass for the j'th mode and $A_0^{(j)}$ is a (complex) normalization constant.

Let us now apply the lifting procedure described above. For the twisted lifting without jumps, with $\overline{U}_0 = \overline{U}_1 = 1$, we deduce $U_0 = U(\theta_0, \phi_0) = U(-\pi/2, -\varphi_0)$ and $U_1 = U(\theta_1, \phi_1) = U(-\pi/2, -\varphi_\pi)$, which implies

$$U_0 = \begin{pmatrix} 0 & -e^{-i\varphi_0} \\ e^{i\varphi_0} & 0 \end{pmatrix}, \qquad U_\Delta = \begin{pmatrix} e^{i(\varphi_0 - \varphi_\pi)} & 0 \\ 0 & e^{i(\varphi_\pi - \varphi_0)} \end{pmatrix}. \tag{3.45}$$

From eq. (3.21) we find $\vec{q}_{(n)} = \vec{q} = (0, 0, 1)$. Lifting the fields from $[0, \pi R]$ to $\mathbb{R}$, using

$$\Psi(-z) = U_0 Z \Psi(z), \qquad \Psi(z + 2n\pi R) = (U_\Delta)^n \Psi(z), \tag{3.46}$$

it is not hard to show that the $b_{1,2}^{(j)}(z)$ are given by eq. (3.43) *for all* $z \in \mathbb{R}$. In particular, they are continuous, so $\overline{U}_n = 1$ and $\widehat{\alpha}_n = 0$. However, because of the nontrivial twists U_n,

$$U_n = U(\theta_n, \phi_n) = U\left(-\frac{\pi}{2}, n(\varphi_0 - \varphi_\pi) - \varphi_0\right), \tag{3.47}$$

the $\alpha_n^\pm$ are nonzero, and given by $\alpha_n^\pm = \mathcal{A}(U_n^{-1}) = e^{-i\phi_n}$.

For the periodic lifting with jumping fields, in which case $U_0 = U_1 = 1$, we apply

$$\Psi(-z) = Z\Psi(z), \qquad \Psi(z + 2n\pi R) = \Psi(z). \tag{3.48}$$

Using eq. (3.43) for $z \in [0, \pi R]$, we can write explicit expressions for the lifted fields for any $z \in \mathbb{R}$. For the jump matrices, we find

$$\overline{U}_0 = \overline{U}_{2k} = U\left(\frac{\pi}{2}, -\varphi_0\right), \qquad \overline{U}_\pi = \overline{U}_{2k-1} = U\left(-\frac{\pi}{2}, -\varphi_\pi\right), \tag{3.49}$$

so $\widehat{\alpha}_{2k} = e^{i\varphi_0}$ and $\widehat{\alpha}_{2k-1} = -e^{i\varphi_\pi}$. Since there are no twists, we have $\alpha_n^\pm = \pm\widehat{\alpha}_n$, as well as $\vec{q}_{2k} = (0, 0, 1)$ and $\vec{q}_{2k-1} = (0, 0, -1)$. For this lifting, the warp factors $b_{1,2}^{(j)}(z)$ are of definite parity: $b_1^{(j)}(z)$ is even and continuous, while $b_2^{(j)}(z)$ is odd and jumping. The two cases for this example are illustrated in figure 1.

3.4 Summary

In this paper, we studied supergravity on a slice of AdS_5. Using the "downstairs" picture, we presented locally supersymmetric boundary conditions that support warped backgrounds. For every bosonic background, we found a set of fermionic boundary conditions (α_0, α_π) that preserve global supersymmetry. For every AdS_4 background, we also found a continuous family of boundary conditions that spontaneously break global supersymmetry. The set is parametrized by the phase difference $\Delta\varphi = \varphi_0 - \varphi_\pi$, where the φ_i are the phases of α_i when $q_3 = 1$.

The "flipped" boundary conditions of refs. [5, 7] correspond to the case $\Delta\varphi = \pi$. In this paper we generalized the flip to a continuous set of supersymmetry-breaking boundary conditions. The mass shift for the Kaluza-Klein fermionic modes depends on $\Delta\varphi$ and can be turned continuously on and off.

The spontaneous supersymmetry breaking discussed here occurs only in AdS_4 backgrounds. When the brane tensions are tuned, as in the original Randall-Sundrum scenario, the boundary conditions are fixed *uniquely*. (For example, eq. (2.27) requires that $\alpha_0 = 0$

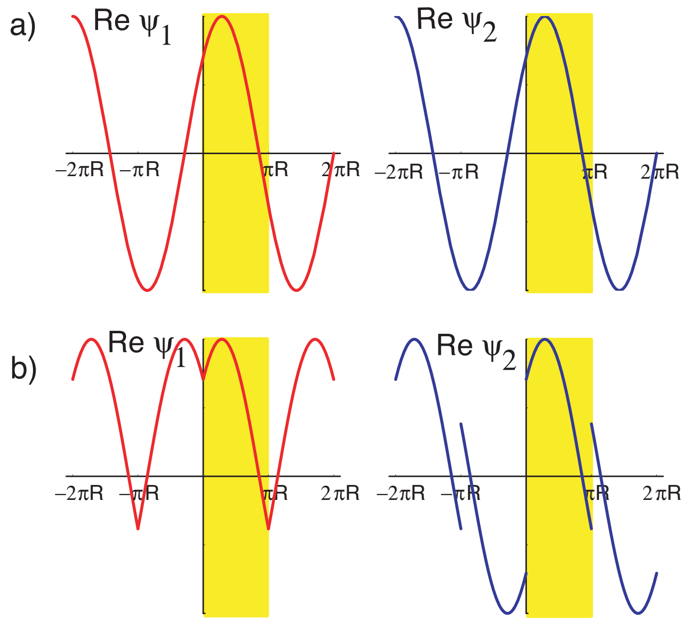

Figure 1: Two gravitino modes, from the example in section 3.3.5. The physics is determined by the fundamental domain, shaded. a) A continuous, but not $2\pi R$-periodic, lifting to $\mathbb{R}$. b) A $2\pi R$-periodic, but not continuous, lifting to $\mathbb{R}$. With this lifting, ψ_1 is even, and ψ_2 is odd.

when $q_3 = 1$ and $\lambda_0 = \lambda$.) Furthermore, the boundary conditions are such that a Killing spinor always exists, and supersymmetry is not broken. This agrees with the conclusion of ref. [10].

The "upstairs" picture is based on a lifting of the physical (fundamental) domain. The lifting uses a *broken* symmetry group G, a combination of rotations on the fields and motions on the covering space. The symmetry is such that it can be restored by rotating the parameters as spurions. It can also be restored by choosing different parameters on the different domains, consistent with the group G, which maps one domain to another. We take the second approach, and choose the parameters so that the lifted theory is G-invariant (*without* rotating the parameters).

The lifting from $[0, \pi R]$ to S^1 uses $G = \mathbb{Z}_2$, generated by the reflection $x^5 \to -x^5$ together with a parity transformation on the fields. The Scherk-Schwarz lifting to $\mathbb{R}$ uses a twisted parity on the fermionic fields. In this case $G = \mathbb{Z}_2 \times \mathbb{Z}$ (a semi-direct product).

In each case, the physics is uniquely specified by the boundary conditions on the fundamental domain. The lifting, however, is not unique. There is freedom to choose the twist and jump parameters. In fact, one can construct twisted lifting even when the boundary conditions (α_0, α_π) do not break supersymmetry.

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